**Decision Theory**

* Decision rule: maximum expected utility for each action (ex. 0.7 \* 10) + (0.3 \* 1) = 7.3
* Breaks down because other agent may influence result
* Substitutability: indifference in outcomes implies indifference in actions (if o1 and o2 give same result, doesn’t matter which one you choose)
* Decomposability: actions with the same probability (utility is defined elsewhere) are equivalent (indifference in outcome)
* Monotonicity: you prefer action A over action B because action A gives you a higher chance that you get the outcome you want.
* Continuity: If A’s probability becomes too low, you’ll prefer B over A.

**Game Theory**

* Point of departure = interactive decision theory
* Game form: (N, A, O, g) where N = players, A = strategy profile space (A1 x … x An), O = outcomes, g : A 🡪 O = outcome function
* Strategic game: game form + u = set of utility functions, one for each player
* Assume A = O to write (N, A, u)
* ui(a) = utility of strategy a for agent i

**Mixed Strategies**

* Mixed strategy = distribution of probability over a strategy
* Pure strategy: go 100% for a single action
* Δ (Ai) = set of mixed strategy profiles, e.g. { s1 = (0.3, 0.7), s2 = (0.5, 0.5) }, where probabilities distribute over the actions, e.g. s1 🡪 0.3 : action1, 0.7 : action2
* Expected utility calculation: see slides. Idea is to take each outcome and multiply it by probability of the action for all players. E.g. if player A does action1 with probability 0.3 and player B does action1 with prob. 0.6, and the outcome of both players doing action1 is 5 for player A, then the utility of that outcome is: 5 \* 0.3 \* 0.6. Do this for each outcome and sum everything.
* Pareto efficiency: An outcome is Pareto efficient if there is no other outcome that makes the outcome better for both players. **Note:** This means there can be multiple Pareto efficient outcomes: (2,2) trumps (1,1) and both (0,3) and (3,0) trump (2,2).  
  **Trick:** Make a graph, x = outcome values for row, y = outcome values for column, map outcomes accordingly. Connect most outer outcomes: these are Pareto efficient.
* Pareto efficient mixed strategy profile = no other mixed strategy profile (so a list of profiles for each player) that is strictly better for all players.
* Pareto dominance: A pure strategy strat1 for a player dominates another strategy strat2 if, for all strategies of the opponents, strat1 leads to a more preferable outcome.  
  Example:

|  |  |  |
| --- | --- | --- |
|  | A | B |
| A | 2,2 | 0,3 |
| B | 3,0 | 1,1 |

The strategy B is strongly dominant for both players because it yields the best results regardless of what the other player may do.

* Mixed strategy strat1 dominates another mixed strategy strat2 if for all mixed strategies of the opponents, utility of strat1 is better than strat2.
* Strongly dominant strategy = mixed strategy of a player that dominates all other mixed strategies of that player
* Dominant mixed strategy equilibrium: every player has a strongly dominant strategy.