**Decision Theory**

* Decision rule: maximum expected utility for each action (ex. 0.7 \* 10) + (0.3 \* 1) = 7.3
* Breaks down because other agent may influence result
* Substitutability: indifference in outcomes implies indifference in actions (if o1 and o2 give same result, doesn’t matter which one you choose)
* Decomposability: actions with the same probability (utility is defined elsewhere) are equivalent (indifference in outcome)
* Monotonicity: you prefer action A over action B because action A gives you a higher chance that you get the outcome you want.
* Continuity: If A’s probability becomes too low, you’ll prefer B over A.

**Game Theory**

* Point of departure = interactive decision theory
* Game form: (N, A, O, g) where N = players, A = strategy profile space (A1 x … x An), O = outcomes, g : A 🡪 O = outcome function
* Strategic game: game form + u = set of utility functions, one for each player
* Assume A = O to write (N, A, u)
* ui(a) = utility of strategy a for agent i

**Mixed Strategies**

* Mixed strategy = distribution of probability over a strategy
* Pure strategy: go 100% for a single action
* Δ (Ai) = set of mixed strategy profiles, e.g. { s1 = (0.3, 0.7), s2 = (0.5, 0.5) }, where probabilities distribute over the actions, e.g. s1 🡪 0.3 : action1, 0.7 : action2
* Expected utility calculation: see slides. Idea is to take each outcome and multiply it by probability of the action for all players. E.g. if player A does action1 with probability 0.3 and player B does action1 with prob. 0.6, and the outcome of both players doing action1 is 5 for player A, then the utility of that outcome is: 5 \* 0.3 \* 0.6. Do this for each outcome and sum everything.
* Pareto efficiency: An outcome is Pareto efficient if there is no other outcome that makes the outcome better for both players. **Note:** This means there can be multiple Pareto efficient outcomes: (2,2) trumps (1,1) and both (0,3) and (3,0) trump (2,2).  
  **Trick:** Make a graph, x = outcome values for row, y = outcome values for column, map outcomes accordingly. Connect most outer outcomes: these are Pareto efficient.
* Pareto efficient mixed strategy profile = no other mixed strategy profile (so a list of profiles for each player) that is strictly better for all players.
* Pareto dominance: A pure strategy strat1 for a player dominates another strategy strat2 if, for all strategies of the opponents, strat1 leads to a more preferable outcome.  
  Example:

|  |  |  |
| --- | --- | --- |
|  | strat2 | strat1 |
| strat2 | 2,2 | 0,3 |
| strat1 | 3,0 | 1,1 |

Row player wants to move to strat2? No, may make it worse.

Column player wants to move to strat2? No, may make it worse.

The strategy B is strongly dominant for both players because it yields the best results regardless of what the other player may do.

* Mixed strategy strat1 dominates another mixed strategy strat2 if for all mixed strategies of the opponents, utility of strat1 is better than strat2.
* Strongly dominant strategy = mixed strategy of a player that dominates all other mixed strategies of that player
* Dominant mixed strategy equilibrium: every player has a strongly dominant strategy.
* To check for Nash: check if row player wants to deviate. If no, check if column player wants to deviate. If no, Nash. All other cases: not Nash.

**Extensive Games**

* Turn-based
* From Perfect-Information extensive to normal-form: Take actions P1 can do at all nodes he controls, do Cartesian product. Example: P1 = {A,B},{C,D} 🡪 {AC,BC,BC,BD}
* Filling in normal-form: take strategy profile (ex. (AC, A’C’ (column player)) ), follow the paths and fill in utility from the graph.
* Equilibria are preserved through this isomorphism.
* TODO: Explain subgame-perfect equilibrium
* Can’t go from all normal-form to extensive, may change equilibria
* This is possible with Imperfect-Information extensive games.
* Imperfect-Information extensive creation: make exercises
* Imperfect-Information extensive to Normal Form: discrepancies arise
* Generate behavioural strategy for player: for each state player can reach: {utility for player} \* {probability of actions taken **by player** to reach utility, multiplied by each other}. Sum all.  
  Example: Slide 26: 1 \* (prob. L) \* (prob. L) + 100 \* (prob. L) (prob. R) + 2 \* (prob. R)

**Communication**

**Cheap Talk**

* Costless, not necessarily truthful, does not imply commitment
* Not worthless! Can change belief about other player’s action, selecting one equilibrium out of multiple equilibria.
* Self-committing utterance: If I assume the other player will believe it, the declared action is the optimal one.
* Self-revealing utterance: If I assume the other player said something because he thought I’d believe it, he only said it because he was going to act that way.